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Functional Modelling of Musical Harmony

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This talk

- Modelling musical harmony using Haskell

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- From textual chord labels to analysis trees

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- Avoiding programming

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- Modelling musical harmony using Haskell
- Modelling music~~al harmony~~ using Haskell
- From textual chord labels to analysis trees
- Avoiding programming
- Why Haskell is a better fit than Java for harmony modelling

Outline

Harmony

Basic musical notions in Haskell

A harmony model in Haskell

From chord text to harmonic structure

Increasing model complexity

Conclusion

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What is harmony?

Ton *SDom* *Dom* *Ton*

I IV V/V V I

C F D^7 G^7 C

- Harmony arises when at least two notes sound at the same time
- Harmony induces tension and release patterns, that can be described by music theory and music cognition
- The internal structure of the chord has a large influence on the consonance or dissonance of a chord
- The surrounding context also has a large influence

Simplified harmony theory I

- A chord is a group of tones separated by intervals of roughly the same size.
- All music is made out of chords (whether explicitly or not).
- There are 12 different notes. Instead of naming them, we number them relative to the first and most important one, the tonic. So we get I, II♭, II . . . VI♯, VII.
- A chord is built on a root note. So I also stands for the chord built on the first degree, V for the chord built on the fifth degree, etc.
- So the following is a chord sequence: I IV II^7 V^7 I.

Simplified harmony theory II

Models for musical harmony explain the harmonic progression in music:

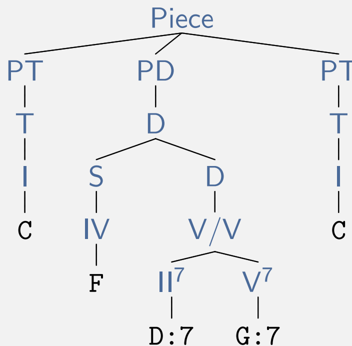
- Everything works around the tonic (I).
- The dominant (V) leads to the tonic.
- The subdominant (IV) tends to lead to the dominant.
- Therefore, the I IV V I progression is very common.
- There are also secondary dominants, which lead to a relative tonic. For instance, II^7 is the secondary dominant of V, and I^7 is the secondary dominant of IV.
- So you can start with I, add one note to get I^7, fall into IV, change two notes to get to II^7, fall into V, and then finally back to I.

An example harmonic analysis

Ton *SDom* *Dom* *Ton*

I IV V/V V I

C F D^7 G^7 C



Why are harmony models useful?

Having a model for musical harmony allows us to automatically determine the functional meaning of chords in the tonal context.

The model determines which chords "fit" on a particular moment in a song.

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Having a model for musical harmony allows us to automatically determine the functional meaning of chords in the tonal context.

The model determines which chords “fit” on a particular moment in a song. This is useful for:

- Musical information retrieval (find songs similar to a given song)
- Audio and score recognition (improving recognition by knowing which chords are more likely to appear)
- Music generation (create sequences of chords that conform to the model)

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Pure
: There are no side-effects, so Haskell functions are like mathematical functions.

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Pure There are no side-effects, so Haskell functions are like mathematical functions.

Functional A Haskell program is an expression, not a sequence of statements. Functions are first class citizens, and explicit state is avoided.

Notes

```
data Root    = A | B | C | D | E | F | G
type Octave = Int
data Note    = Note Root Octave
```

Notes

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data Root    = A | B | C | D | E | F | G
type Octave = Int
data Note    = Note Root Octave

a4, b4, c4, d4, e4, f4, g4 :: Note
a4 = Note A 4
b4 = Note B 4
c4 = Note C 4
d4 = Note D 4
e4 = Note E 4
f4 = Note F 4
g4 = Note G 4
```

Melody

```
type Melody = [Note]

cMajScale :: Melody
cMajScale = [c4, d4, e4, f4, g4, a4, b4]
```

Melody

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cMajScale :: Melody
cMajScale = [c4, d4, e4, f4, g4, a4, b4]

cMajScaleRev :: Melody
cMajScaleRev = reverse cMajScale
```

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Melody

```
type Melody = [Note]

cMajScale :: Melody
cMajScale = [c4, d4, e4, f4, g4, a4, b4]

cMajScaleRev :: Melody
cMajScaleRev = reverse cMajScale

reverse :: [α] → [α]
reverse []      = []
reverse (h : t) = reverse t ++ [h]
(++) :: [α] → [α] → [α]
(++) = ...
```

Transposition

Transposing a melody one octave higher:

```
octaveUp :: Octave → Octave
octaveUp n = n + 1

noteOctaveUp :: Note → Note
noteOctaveUp (Note r o) = Note r (octaveUp o)

melodyOctaveUp :: Melody → Melody
melodyOctaveUp m = map noteOctaveUp m
```

Generation, analysis

Building a canon from a melody:

$$\text{canon} :: \text{Melody} \rightarrow \text{Melody}$$
$$\text{canon}\ m = m \mathbin{+\!\!+} \text{canon}\ m$$

Generation, analysis

Building a canon from a melody:

$$\text{canon} :: \text{Melody} \to \text{Melody}$$
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Is a given melody in C major?

$$\text{root} :: \text{Note} \to \text{Root}$$
$$\text{root } (\text{Note } r \; o) = r$$

$$\text{isCMaj} :: \text{Melody} \to \text{Bool}$$
$$\text{isCMaj} = (\equiv \text{map root cMajScaleRev}) \circ \text{sort} \circ \text{nub} \circ \text{map root}$$

“Details” left out

We have seen only a glimpse of music representation in Haskell.

- Rhythm
- Accidentals
- Intervals
- Voicing
- ...

A good pedagogical reference on using Haskell to represent music: `http://di.uminho.pt/~jno/html/ipm-1011.html`

A serious library for music manipulation: `http://www.haskell.org/haskellwiki/Haskore`

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Representing harmony as a datatype, naively

$$
\begin{array}{llllll}
\textbf{data}\ \text{Piece} & = \text{Piece} & [\text{Phrase}] & & & \\
\textbf{data}\ \text{Phrase} & = \text{PT} & \text{Ton} & \mid \text{PD} & \text{Dom} & \\
\textbf{data}\ \text{Ton} & = \text{T}_{\text{IMaj}} & \text{DegNum} & & & \\
\textbf{data}\ \text{Dom} & = \text{D}_{\text{VMaj}} & \text{DegNum} & \mid \text{D}_{\text{SDom}} & \text{SDom} & \text{Dom} \\
\textbf{data}\ \text{SDom} & = \text{S}_{\text{IVMaj}} & \text{DegNum} & & & \\
\textbf{data}\ \text{DegNum} & = \text{I} \mid \text{II} \mid \text{III} \ldots & & & &
\end{array}
$$

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Representing harmony as a datatype, naively

```
data Piece  = Piece [Phrase]
data Phrase = PT Ton | PD Dom
data Ton    = T_IMaj DegNum
data Dom    = D_VMaj DegNum | D_SDom SDom Dom
data SDom   = S_IVMaj DegNum
data DegNum = I | II | III ...
```

Problem: the term Piece [PT (T_{IMaj} II)] typechecks, but represents an invalid harmonic structure. T_{IMaj} should only take I as argument.

How can we enforce this? (And why do we want it?)

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Representing harmony as a datatype I

```
data Piece  = Piece    [Phrase]
data Phrase = PT       Ton        | PD Dom
data Ton    = T_IMaj   (Deg I)
data Dom    = D_VMaj   (Deg V)    | D_SDom SDom Dom
data SDom   = S_IVMaj  (Deg IV)
```

```
data Deg δ = Deg DegNum
```

We do not export the Deg constructor. So how do we build Degs?

Representing harmony as a datatype II

$$\mathsf{deg} :: \mathsf{ToDegree}\ \delta \Rightarrow \delta \rightarrow \mathsf{Deg}\ \delta$$
$$\mathsf{deg}\ \mathsf{d} = \mathsf{Deg}\ (\mathsf{toDegree}\ \mathsf{d})$$

We need degrees at the type level:

```
data I; data II; ... data VII;
```

And type-to-value conversions for degrees:

```
class    ToDegree δ  where toDegree :: δ → DegNum
instance ToDegree I  where toDegree _ = I
instance ToDegree II where toDegree _ = II
...   -- instances for all degrees
```

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Representing harmony as a datatype III

Now the term Piece [PT (T_{IMaj} (deg ($\bot$ :: II)))] does not typecheck, as we wanted:

```
*ghci> Piece [PT (TIMaj (deg (undefined :: II)))]

    Couldn't match expected type ‘I’
      with actual type ‘II’
```

Representing harmony as a datatype IV

We have a well-typed harmony model in Haskell that accepts only "correct" harmonic sequences.

Here we have only seen a very basic subset of our model. In reality, it consists of 14 datatypes with a total of 46 constructors.

Also, while developing the model we have to change it often, trying to find a balance between complexity and expressiveness.

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Parsing chord sequences I

We now want to parse (textual) chord sequences into our datatype representing musical harmony:

```
class Parse α where
  parse :: ParserMusic α
```

Parsing chord sequences II

Most instances are trivial:

$$
\begin{aligned}
&\textbf{instance } \mathsf{Parse}\ \mathsf{Phrase}\ \textbf{where}\\
&\quad \mathsf{parse} = \mathsf{PT} <\$> \mathsf{parse}\\
&\qquad\quad <|> \mathsf{PD} <\$> \mathsf{parse}\\
&\textbf{instance } \mathsf{Parse}\ \mathsf{Ton}\ \textbf{where}\\
&\quad \mathsf{parse} = \mathsf{T}_{\mathsf{IMaj}} <\$> \mathsf{parse}\\
&\textbf{instance } \mathsf{Parse}\ \mathsf{Dom}\ \textbf{where}\\
&\quad \mathsf{parse} = \mathsf{D}_{\mathsf{VMaj}} <\$> \mathsf{parse}\\
&\qquad\quad <|> \mathsf{D}_{\mathsf{SDom}} <\$> \mathsf{parse} <*> \mathsf{parse}\\
&\ldots
\end{aligned}
$$

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Parsing chord sequences II

Most instances are trivial:

```
instance Parse Phrase where
  parse = PT <$> parse
      <|> PD <$> parse
instance Parse Ton where
  parse = T_IMaj <$> parse
instance Parse Dom where
  parse = D_VMaj <$> parse
      <|> D_SDom <$> parse <*> parse
...
```

So trivial that we do not write them; we use a generic parser.

Parsing degrees

For degrees we need an adhoc parser:

$$\textbf{instance}\ (\text{ToDegree}\ \delta) \Rightarrow \text{Parse}\ (\text{Deg}\ \delta)\ \textbf{where}$$
$$\quad \text{parse} = \text{pChord}\ (\text{toDegree}\ (\bot :: \delta))$$

$$\text{pChord} :: \text{DegNum} \rightarrow \text{ParserMusic}\ (\text{Deg}\ \delta)$$
$$\text{pChord} = \ldots$$

Again we use the type-to-value conversion class ToDegree.

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Increasing complexity

The model we presented so far is very simple and cannot account for:

Repeated chords $\underline{G_{Maj}\ G_{Maj}}\ C_{Maj}$

Diatonic secondary dominants $\underline{E_{min}\ A_{min}\ D_{min}}\ G^7\ C_{Maj}$

Chromatic secondary dominants $\underline{A^7\ D^7}\ G^7\ C_{Maj}$

Minor key dominant borrowing $\underline{G_{min}}\ C_{Maj}$

Tritone substitutions $\underline{A\flat^7}\ G^7\ C_{Maj}$

...

Handling repeated chords

Easy: adapt the Deg type and pChord:

```
data Deg δ = Deg {chordRole :: DegNum
                 , numReps   :: Int}

pChord :: DegNum → ParserMusic (Deg δ)
pChord = . . .    -- adapted accordingly
```

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Secondary dominants—datatype representation

Here the fun begins! We adapt the Deg type again:

$$\textbf{type}\ \mathsf{Deg}_{\mathsf{SD}}\ \delta = \mathsf{BDeg}_{\mathsf{SD}}\ \delta$$

$$\begin{aligned}&\textbf{data}\ \mathsf{BDeg}_{\mathsf{SD}}\ \delta\ \textbf{where}\\ &\quad \mathsf{Cons} :: \mathsf{BDeg}_{\mathsf{SD}}\ (\mathsf{Perf}_{\mathsf{V}}\ \delta) \to \mathsf{Deg}\ \delta \to \mathsf{BDeg}_{\mathsf{SD}}\ \delta\\ &\quad \mathsf{Base} :: \mathsf{Deg}\ \delta \to \mathsf{BDeg}_{\mathsf{SD}}\ \delta\end{aligned}$$

Secondary dominants—datatype representation

Here the fun begins! We adapt the Deg type again:

$$\textbf{type}\ \mathsf{Deg}_{\mathsf{SD}}\ \delta = \mathsf{BDeg}_{\mathsf{SD}}\ \delta$$

$$\textbf{data}\ \mathsf{BDeg}_{\mathsf{SD}}\ \delta\ \textbf{where}$$

$$\quad \mathsf{Cons} :: \mathsf{BDeg}_{\mathsf{SD}}\ (\mathsf{Perf}_{\mathsf{V}}\ \delta) \rightarrow \mathsf{Deg}\ \delta \rightarrow \mathsf{BDeg}_{\mathsf{SD}}\ \delta$$

$$\quad \mathsf{Base} :: \mathsf{Deg}\ \delta \rightarrow \mathsf{BDeg}_{\mathsf{SD}}\ \delta$$

$$\textbf{data}\ \mathsf{Deg}\ \delta = \mathsf{Deg} \ldots \quad \text{-- as before}$$

$$\textbf{type family}\ \mathsf{Perf}_{\mathsf{V}}\ \delta \quad \text{-- computes the perfect fifth}$$

$$\textbf{type instance}\ \mathsf{Perf}_{\mathsf{V}}\ \mathsf{I} = \mathsf{V}$$

$$\textbf{type instance}\ \mathsf{Perf}_{\mathsf{V}}\ \mathsf{V} = \mathsf{II}$$

$$\ldots$$

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Example

Parsing the sequence G_{min} C^7 G_{min} C^7 F_{Maj} D^7 G^7 C_{Maj}:

- Piece
  - PD
    - D
      - S
        - V/IV
          - V/I
            - V_{min}
              - `G:min`
          - I^7
            - `C:7`
        - IV
          - `ins`
      - D
        - S
          - V/IV
            - V/I
              - V_{min}
                - `G:min`
            - I^7
              - `C:7`
          - IV
            - `F:maj`
        - D
          - V/V
            - II^7
              - `D:7`
          - V^7
            - `G:7`
  - PT
    - T
      - I
        - `C:maj`

Harmonic similarity

- A practical application of a harmony model is to estimate harmonic similarity between songs
- The more similar the trees, the more similar the harmony
- We don't want to write a diff algorithm for our complicated model; we get it automatically by using a generic diff
- The generic diff is a type-safe tree-diff algorithm. It is part of Eelco Lempsink's MSc work here in Utrecht
- Generic, thus working for any model, and independent of changes to the model

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Comparison to a Java approach

Comparing the current solution to an earlier Java approach we find that:

- Error-correction makes the grammar simpler, and ensures parsing never fails
- GADTs allow us to control grammar ambiguity
- GADTs help simplifying the grammar
- The Haskell parser is faster and more robust
- The generic diff performs well
- The Haskell approach has fewer source lines of code

Summary

What we have so far:

- A model for musical harmony as a Haskell GADT
- The datatypes can change, the code does not have to:
  - Generic parser
  - Generic pretty-printer
  - Generic diff
- Multiple models, lots of code reuse
- When chords do not fit the model: error correction
- Recognizing harmony from audio sources

Play with it!

http://hackage.haskell.org/package/HarmTrace