

Generic Generation of Constrained Random Data

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Being able to generate random data is useful:

► Testing



- ► Testing
- Examples



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- ► In complex domains:



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- ▶ In complex domains:
 - Exercises in an exercise assistant



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 - Population for genetic/evolutionary algorithms



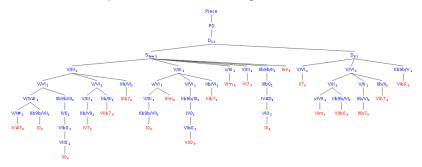
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Generic random data generation: why?



Being able to generate random data generically is useful too:

 Data generators can be tedious to write; if they're generic, we don't have to write them.

Generic random data generation: why?



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- Datatypes change, and the code to generate them must change too—unless it's generic.

Generic random data generation: why?



Being able to generate random data *generically* is useful too:

- Data generators can be tedious to write; if they're generic, we don't have to write them.
- Datatypes change, and the code to generate them must change too—unless it's generic.
- ► Good generators can be hard to get right; they have to take into account the size of the generated terms, ensure a fair distribution, etc. If we do it generically, the hard work is done "once and for all".

Generic, but constrained, random data generation



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Generic, but constrained, random data generation



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Unfortunately, data generation is very often tied to semantics, especially for complex data:

- ► Generating sorted lists
- ► Generating balanced trees
- Generating good exercises (not too easy, not too hard)
- Generating valid programs

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- Generating good exercises (not too easy, not too hard)
- Generating valid programs

What we need is a hybrid solution, where generic programming does the "boring parts", but still allows us to specify constraints on the shape of the data generated.

Easy way: generate random data, then filter



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Assuming we can just generically generate random data, we could then simply *filter* the data according to some predicate.

In practice, however, this can be terribly inefficient, especially if the interesting data is a very small subset of all possible data. So we don't want to take this approach (but see Claessen et al., "Generating Constrained Random Data with Uniform Distribution" @ FLOPS'14).



We're aiming at something relatively complicated, so we'll go step-by-step:

1. Generating random data generically, no constraints



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- 2. Constraining constructor frequency



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- 3. Constraining size and order
- 4. Constraining constructor count

What we will not see:

- ► Efficiency concerns
- ► Nested datatypes, GADTs, etc.

Generic programming support: universe



We need a library for generic programming:

```
kind ArgType = Par \mid Rec RecType \mid Unknown
kind RecType = Self \mid Other
```

Generic programming support: to/from



Witnessing the isomorphism between a datatype and its generic representation:

```
class Generic (\alpha :: \star) where type Rep \alpha :: Univ from :: \alpha \to [\![Rep \ \alpha \ ]\!] to :: [\![Rep \ \alpha \ ]\!] \to \alpha
```

Generic programming support: to/from



Witnessing the isomorphism between a datatype and its generic representation:

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class Generic (\alpha :: \star) where type Rep \ \alpha :: Univ from :: \alpha \to [Rep \ \alpha] to :: [Rep \ \alpha] \to \alpha
```

For example, lists:



Section 1

Generic data generation, no constraints

Generic data generation, no constraints: I



We need a user-facing class (just like *Arbitrary*):

class Generate α where gen:: Gen α

Generic data generation, no constraints: I



```
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```

```
class Generate \alpha where gen:: Gen \alpha
```

And one to define data generation generically:

```
class GGenerate (v:: Univ) where gGen:: Gen (\llbracket v \rrbracket)
```

Generic data generation, no constraints: II



Now we can define the generic instances:

```
instance (GGenerate v) \Rightarrow GGenerate (Data v) where
  gGen = fmap D gGen
instance (GGenerate \phi, GGenerate \psi) \Rightarrow GGenerate (\phi:+: \psi) where
  gGen = choose [fmap Left gGen, fmap Right gGen]
instance (GGenerate v) \Rightarrow GGenerate (Con v) where
  gGen = fmap C gGen
instance GGenerate Unit where
  gGen = return Unit
instance (GGenerate \phi, GGenerate \psi) \Rightarrow GGenerate (\phi:×: \psi) where
  gGen = (:\times:) < $> gGen < *> gGen
instance (Generate \alpha) \Rightarrow GGenerate (K \iota \alpha) where
  gGen = fmap K gen
```

Generic data generation, no constraints: III



Finally, a generic dispatcher:

```
genDefault :: (Generic \ \alpha, GGenerate \ (Rep \ \alpha)) \Rightarrow Gen \ \alpha genDefault = fmap to gGen
```

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genDefault :: (Generic \ \alpha, GGenerate \ (Rep \ \alpha)) \Rightarrow Gen \ \alpha genDefault = fmap to gGen
```

And we're ready to generically generate random, unconstrained data.

```
> sample (genDefault :: [Bool])
[]
[True, False, True]
[]
[False]
[]
[True, True]
[]
[False]
```



Section 2

Constructor frequency constraints

Constructor frequency constraints: I



Now let's start considering constraints. First, we want to be able to say that some constructors should occur more/less often than others.

We will need to keep track of some information during generation:

Constructor frequency constraints: II



We adapt the type of our functions:

```
class Generate \alpha where gen :: MyState \rightarrow Gen \alpha class GGenerate (v :: Univ) where gGen :: MyState \rightarrow Gen ([v])
```

Constructor frequency constraints: II



We adapt the type of our functions:

class Generate \alpha where

```
gen :: MyState \rightarrow Gen \alpha
     class GGenerate (v:: Univ) where
        gGen :: MvState \rightarrow Gen (\llbracket \upsilon \rrbracket)
And each of the generic cases:
     instance (Singl 1, GGenerate v) \Rightarrow GGenerate (Data 1 v) where
        gGen s = fmap D (gGen (s \{ dtNameState = fromSing (sing :: Sing t) \}))
     instance (GGenerate v) \Rightarrow GGenerate (Con v) where
        gGen = fmap C \circ gGen
     instance GGenerate Unit where
        gGen = return Unit
     instance (GGenerate \phi, GGenerate \psi) \Rightarrow GGenerate (\phi:×: \psi) where
        gGen s = (:\times:) < \$ > gGen s < * > gGen s
     instance (Generate \alpha) \Rightarrow GGenerate (K \iota \alpha) where
        gGen = fmap K \circ gen
```

Constructor frequency constraints: III



The interesting case is that of sums:

```
instance (GGenerate \alpha, GConNames \alpha
          , GGenerate \beta, GConNames \beta) \Rightarrow GGenerate (\alpha:+: \beta) where
  gGen s = let \ aNames = gconNames (Proxy :: Proxy ( [ \alpha ] ) )
                 bNames = gconNames (Proxy :: Proxy ([B]))
                 dtName = dtNameState s
                 env = envState s
                 aFreq = frequencies aNames dtName env
                 bFreq = frequencies bNames dtName env
             in frequency [( aFreq, fmap Left (gGen s))
                           ( bFreq, fmap Right (gGen s))]
frequencies :: [String] \rightarrow String \rightarrow Env \rightarrow Int
frequencies []
frequencies (s:ss) dtName env =
  let freqS = maybe\ 1 freqInfo\ (lookup\ (dtName, s)\ env)
  in freqS + frequencies ss dtName env
frequency :: [(Int, Gen \alpha)] \rightarrow Gen \alpha
```

Constructor frequency constraints: IV



We need to be able to collect the names of all constructors of a datatype:

```
data Proxv \alpha = Proxv
class GConNames (v:: Univ) where
  gconNames :: Proxy([v]) \rightarrow [String]
instance (GConNames v) \Rightarrow GConNames (Data v) where
  gconNames \_ = gconNames (Proxy :: Proxy ([v]))
instance (GConNames \phi, GConNames \psi) \Rightarrow GConNames (\phi:+: \psi) where
  gconNames \_ = gconNames (Proxy :: Proxy ( \llbracket \phi \rrbracket ) )
                     ++ gconNames (Proxy :: Proxy (\llbracket \psi \rrbracket))
instance (Singl 1) \Rightarrow GConNames (Con 1 v) where
  gconNames = [fromSing (sing :: Sing t)]
class ConNames (\alpha :: \star) where
  conNames :: Proxy \alpha \rightarrow [String]
```

Constructor frequency constraints: V



Now we can constrain constructor frequency!

```
data Choice = A \mid B \mid C \mid D

testChoice :: IO ()

testChoice = let s = MyState (fromList reqEnv) ""

reqEnv = [(("Choice", "A"), ConInfo 1)

,(("Choice", "B"), ConInfo 2)

,(("Choice", "C"), ConInfo 3)

,(("Choice", "D"), ConInfo 4)]

in testM (genDefault s :: Gen Choice) 1000 >>= mapM_print

testM :: Gen \alpha \rightarrow Int \rightarrow IO [(Int, \alpha)]

-- takes a generator, generates n samples, groups them
```

Constructor frequency constraints: V



Now we can constrain constructor frequency!

```
data Choice = A \mid B \mid C \mid D
testChoice :: IO()
testChoice = let s = MyState (fromList regEnv)""
                 regEnv = [(("Choice", "A"), ConInfo 1)]
                            (("Choice", "B"), ConInfo 2)
                            (("Choice", "C"), ConInfo 3)
                            ,(("Choice", "D"), ConInfo 4)]
              in testM (genDefault s:: Gen Choice) 1000 ≫ mapM print
testM :: Gen \alpha \rightarrow Int \rightarrow IO [(Int, \alpha)]
  -- takes a generator, generates n samples, groups them
> testChoice
(99, A)
(212, B)
(314, C)
(376, D)
```

Constructor frequency constraints: VI



We can even forbid certain constructors, and (indirectly) influence size:

```
testListBool :: IO () \\ testListBool = \textbf{let } s = & \textit{MyState} (fromList \ reqEnv) "" \\ reqEnv = [(("Bool", "True"), ConInfo 1), (("Bool", "False"), ConInfo 0), (("[]", ":"), ConInfo 3), (("[]", "[]"), ConInfo 1)] \\ \textbf{in } testM (genDefault \ s :: Gen [Bool]) 10 >>= mapM_print
```

Constructor frequency constraints: VI



We can even forbid certain constructors, and (indirectly) influence size:

```
testListBool :: 10 ()
testListBool = let s = MyState (fromList reqEnv)""
                 regEnv = [(("Bool", "True"), ConInfo 1)
                           (("Bool", "False"), ConInfo 0)
                          ,(("[]",":") ,ConInfo 3)
                           ,(("[]","[]") ,ConInfo 1)]
              in testM (genDefault s:: Gen [Bool]) 10 ≫ mapM print
> testListBool
(3,[])
(3, [True])
(1, [True, True])
(1, [True, True, True])
(3, [True, True, True, True])
```



Section 3

Size and order constraints

Size and order constraints: I



Another type of constraint is enforcing *order* on the *size* of a specific argument to a constructor. We will need to carry more state:

```
data MyState = MyState  { envState :: Env
                        . dtNameState :: String
                        . ctNameState :: String -- constructor name
                        , argIndexState:: Int } -- argument index
type Env = Map ConLoc ConInfo
type ConLoc = (String, String)
type ArgLoc = Int
data ConInfo = ConInfo { freqInfo :: Int
                        , argInfo :: Map ArgLoc ArgInfo }
data ArgInfo = ArgInfo { ordInfo :: Ordering -- ordering
                       , sizeInfo :: Int } -- size
```

Size and order constraints: I



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type ConLoc = (String, String)
type ArgLoc = Int
data ConInfo = ConInfo { freqInfo :: Int
                        , argInfo :: Map ArgLoc ArgInfo }
data ArgInfo = ArgInfo { ordInfo :: Ordering -- ordering
                       , sizeInfo :: Int } -- size
```

...and now we'll have to update the state during generation!

Size and order constraints: II



Our generator now behaves like a state monad:

```
class Generate \alpha where gen :: MyState \rightarrow (Gen \alpha, MyState)
class GGenerate (v :: Univ) where gGen :: MyState \rightarrow (Gen ([v]), MyState)
```

At every step, we might change the state, returning an updated version.

Size and order constraints: III



The important changes are in the case for constructor arguments:

```
instance (Elements \alpha, Generate \alpha) \Rightarrow GGenerate (K \iota \alpha) where
   gGen s = let info = ... -- lookups
                    arg \ o \ i = (elements (elems \ i), next \ o \ i)
                    next EQ = id
                    next GT = pred
                    next IT = succ
               in case info of
                    Nothing \rightarrow fmapL K (gen s)
                    Just (ai, ci) \rightarrow let(e, i) = arg(ordInfo ai)(sizeInfo ai)
                                       in (fmap K e, updSize s i)
fmapL :: (\alpha \rightarrow \beta) \rightarrow (\alpha, \gamma) \rightarrow (\beta, \gamma)
updSize :: MvState \rightarrow Int \rightarrow MvState
elements :: \alpha \rightarrow Gen \alpha
```

Size and order constraints: IV



Generation of elements constrained by size is best done by *enumeration*. Here's a naive generic enumerator:

```
class GElements (v::Univ) where gelements::Int \rightarrow [\llbracket v \rrbracket] instance GElements Unit where gelements \ 0 = [Unit] gelements \ 0 = [Unit] gelements \ 0 = [Unit] instance (GElements \alpha, GElements \beta) \Rightarrow GElements (\alpha:+:\beta) where gelements \ n = map \ Left \ (gelements \ n) + map \ Right \ (gelements \ n) instance (Elements \alpha) \Rightarrow GElements (K \ \iota \ \alpha) where gelements \ n = map \ K \ (elements \ n)
```

Size and order constraints: V



For products, we consider all possible size distributions:

```
instance (GElements \alpha, GElements \beta) \Rightarrow GElements (\alpha:×:\beta) where gelements n = [l:::r \mid (a,b) \leftarrow split n
, l \leftarrow gelements a
, r \leftarrow gelements b]
split::lnt \rightarrow [(lnt, lnt)]
split n = [(a, n-a) \mid a \leftarrow [0..n]]
```

As usual, we provide a user-facing class, and instances for base types:

```
class Elements \alpha where elements:: Int \rightarrow [\alpha] instance Elements Bool where elements 1 = [False, True] elements \_ = []
```

Size and order constraints: VI



Now we can generate lists of lists of increasing or decreasing length, for example:

```
-- increasing
[[], [False], [True, False], [True, False, False], [True, False, False, False]]
-- decreasing
[[True, True, False, True], [False, False, True], [True, False]]
-- equal
[[False, True], [False, False], [True, False], [True, False], [False, True]]
```

And we can still provide constructor probabilities!

Size and order constraints: VI



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-- equal
[[False,True],[False,False],[True,False],[True,False],[False,True]]
```

And we can still provide constructor probabilities!

Question: how about constraining the order of the arguments themselves, so we can produce *sorted* lists? It's slightly trickier. . .



Section 4

Constructor count constraints



Finally, let us try to constrain constructors to occur a *minimum* or *maximum* number of times:



The interesting case is at the sum. We assume right-nesting, so there's one argument to the left of a sum, and (potentially) multiple to the right:

```
instance (GGenerate \ \alpha, GConNames \ \alpha

, GGenerate \ \beta, GConNames \ \beta) \Rightarrow GGenerate \ (\alpha :+: \beta) where gGens =

let aNames = \dots

dir = pickLR \ (dtNameState \ s) \ aNames \ bNames \ (envState \ s)

aFreq = \mathbf{if} \ dir \equiv GoRight \ \mathbf{then} \ 0 \ \mathbf{else} \ frequencies \dots

bFreq = \mathbf{if} \ dir \equiv GoLeft \ \mathbf{then} \ 0 \ \mathbf{else} \ frequencies \dots

(genL, sL) = fmapL \ Left \ (gGens)

(genR, sR) = fmapL \ Right \ (gGens)

\mathbf{in} \ (frequency \ [(aFreq, genL), (bFreq, genR)], s) \ -- s?
```



For each constructor, we'll check whether we should or shouldn't produce it:

```
data CanGo = Yes | No | DontCare

instance Monoid CanGo where

mempty = DontCare

mappend DontCare x = x

mappend x DontCare = x

mappend No _ = No

mappend _ No = No

mappend Yes Yes = Yes
```



We use this information to decide whether to go left or right:

```
data Dir = GoLeft | GoRight | DoesntMatter
pickLR :: String \rightarrow [String] \rightarrow [String] \rightarrow Env \rightarrow Dir
pickLR dn cnsL cnsR env =
  let goHere cn = case lookup (dn, cn) env of
                        Nothing \rightarrow DontCare
                        Just i \rightarrow if maxInfo i = 0 then No.
                                      else if minInfo i > 0 then Yes
                                           else DontCare
  in case (mconcat (map goHere cnsL), mconcat (map goHere cnsR)) of
     (No, No) \rightarrow DoesntMatter
     (Yes, \_) \rightarrow GoLeft
     (No, \_) \rightarrow GoRight
                 \rightarrow if any (\equiv Yes) (map goHere cnsR) then GoRight
                    else if all (\equiv No) (map goHere cnsR) then GoLeft
                         else DoesntMatter
```



Now we can generate lists of length between 3 and 7 without Falses:

```
[True, True, True]
[True, True, True, True, True]
[True, True, True, True]
```



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... but a number of problems remain:

► We're too eager to reach the requested minimum; if we request lists with a minimum of one [], we only generate empty lists.



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- ▶ It doesn't really work on types like [[Int]]



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```

... but a number of problems remain:

- We're too eager to reach the requested minimum; if we request lists with a minimum of one [], we only generate empty lists.
- ▶ It doesn't really work on types like [[Int]]
- ...or on mutually recursive types in general:

$$data A = A_1 B B | A_2 B$$
$$data B = B_1$$



Section 5

Conclusion



We've seen a first attempt at generic generation of constrained random data, with a number of limitations:

► How to distinguish the inner and outer constructors of a type such as [[Int]]?



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- ▶ Backtracking?



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- Generation and enumeration: can they be unified?



We've seen a first attempt at generic generation of constrained random data, with a number of limitations:

- ► How to distinguish the inner and outer constructors of a type such as [[Int]]?
- ► Backtracking?
- Generation and enumeration: can they be unified?
- Other types of constraints