

# Generic Representations of Tree Transformations

José Pedro Magalhães

Joint work with Jeroen Bransen

Department of Computer Science  
University of Oxford

September 28, 2013

# What are transformations?



Consider an example datatype:

```
data Expr = Var String
          | Const Int
          | Neg Expr
          | Add Expr Expr
```

# What are transformations?

Consider an example datatype:

```
data Expr = Var String
          | Const Int
          | Neg Expr
          | Add Expr Expr
```

Here are some transformations:

- ▶  $Var\ "a" \rightsquigarrow Neg\ (Var\ "a")$

# What are transformations?



Consider an example datatype:

```
data Expr = Var String
          | Const Int
          | Neg Expr
          | Add Expr Expr
```

Here are some transformations:

- ▶  $Var\ "a" \rightsquigarrow Neg\ (Var\ "a")$
- ▶  $Add\ a\ b \rightsquigarrow Add\ b\ a$

# What are transformations?



Consider an example datatype:

```
data Expr = Var String
          | Const Int
          | Neg Expr
          | Add Expr Expr
```

Here are some transformations:

- ▶  $Var\ "a" \rightsquigarrow Neg\ (Var\ "a")$
- ▶  $Add\ a\ b \rightsquigarrow Add\ b\ a$
- ▶  $Add\ a\ (Add\ b\ c) \rightsquigarrow Add\ (Add\ a\ b)\ c$

# What are transformations?



Consider an example datatype:

```
data Expr = Var String
          | Const Int
          | Neg Expr
          | Add Expr Expr
```

Here are some transformations:

- ▶  $Var\ "a" \rightsquigarrow Neg\ (Var\ "a")$
- ▶  $Add\ a\ b \rightsquigarrow Add\ b\ a$
- ▶  $Add\ a\ (Add\ b\ c) \rightsquigarrow Add\ (Add\ a\ b)\ c$
- ▶  $Add\ a\ (Const\ 0) \rightsquigarrow Add\ a\ a$

# What are transformations?

Consider an example datatype:

```
data Expr = Var String
          | Const Int
          | Neg Expr
          | Add Expr Expr
```

Here are some transformations:

- ▶  $Var\ "a" \rightsquigarrow Neg\ (Var\ "a")$
- ▶  $Add\ a\ b \rightsquigarrow Add\ b\ a$
- ▶  $Add\ a\ (Add\ b\ c) \rightsquigarrow Add\ (Add\ a\ b)\ c$
- ▶  $Add\ a\ (Const\ 0) \rightsquigarrow Add\ a\ a$
- ▶  $Neg\ (Neg\ (Neg\ (Neg\ (Neg\ (Neg\ (Neg\ (Const\ 1))))))) \rightsquigarrow Neg\ (Neg\ (Neg\ (Neg\ (Neg\ (Neg\ (Neg\ (Const\ 2)))))))$

# Why do we need to represent transformations?



Large applications need to *transform*, *edit*, or *evolve* data:

**Structure editors** A structure editor is a type of editor that is aware of the underlying structure of the document being edited.



# Why do we need to represent transformations?



Large applications need to *transform, edit, or evolve* data:

**Structure editors** A structure editor is a type of editor that is aware of the underlying structure of the document being edited.

**Exercise assistants** An exercise assistant is a tool to help students understand and apply fundamental concepts in a given domain.

# Why do we need to represent transformations?



Large applications need to *transform, edit, or evolve* data:

**Structure editors** A structure editor is a type of editor that is aware of the underlying structure of the document being edited.

**Exercise assistants** An exercise assistant is a tool to help students understand and apply fundamental concepts in a given domain.

**Incremental computations** To avoid expensive recomputation in unchanged parts of data, an incremental computation keeps track of changes.

# Diff is not enough



Consider the edit script resulting from computing the difference between *Add (Var "a") (Var "b")* and *Add (Var "b") (Var "a")*:

```
Cpy Add $ Cpy Var $ Ins "b" $ Ins Var $  
Cpy "a" $ Del Var $ Del "b" $ End
```

This edit script does not keep track of the fact that the inserted expressions are not “new”, losing adequate sharing between transformations.

# What we need



A representation of transformations that:

- ▶ Is as abstract as possible as to what type of transformations are allowed
- ▶ Keeps track of subexpression sharing
- ▶ Minimises duplication of information

# Three approaches



We present three different approaches to the problem, of varying complexity, flexibility, and user-friendliness:

1. Zipper with state
2. Rewriting
3. Explicit encoding

Different approaches might be better suited for one particular application area or another.

Obviously, our approach is datatype-generic. For presentation purposes we use the regular library:

**instance** *Regular Expr* **where**

**type** *PF Expr* = *K String* :+: *K Int* :+: *I* :+: (*I* :×: *I*)

from (*Var* *s*) = *L* (*K s*)

from (*Const* *i*) = *R* (*L* (*K i*))

from (*Neg* *e*) = *R* (*R* (*L* (*I e*)))

from (*Add* *e*<sub>1</sub> *e*<sub>2</sub>) = *R* (*R* (*R* (*I e*<sub>1</sub> :×: *I e*<sub>2</sub>)))

to (*L* (*K s*)) = *Var* *s*

to (*R* (*L* (*K i*))) = *Const* *i*

to (*R* (*R* (*L* (*I e*)))) = *Neg* *e*

to (*R* (*R* (*R* (*I e*<sub>1</sub> :×: *I e*<sub>2</sub>)))) = *Add* *e*<sub>1</sub> *e*<sub>2</sub>

# 1. Zipper with state



# 1. Zipper with state

Our first approach combines a zipper with state in a monad:

*insert* :: *Maybe Expr*

*insert* = *navigateZS* (*Add* (*Const* 1) (*Var* "a"))\$

**do** *downZS*

*rightZS*

*updateZS Neg*

*insert* ≡ *Just* (*Add* (*Const* 1) (*Neg* (*Var* "a")))



# 1. Zipper with state

Our first approach combines a zipper with state in a monad:

*insert* :: *Maybe Expr*

```
insert = navigateZS (Add (Const 1) (Var "a")) $  
  do downZS  
    rightZS  
    updateZS Neg
```

*insert*  $\equiv$  *Just* (*Add* (*Const* 1) (*Neg* (*Var* "a")))

*delete* :: *Maybe Expr*

```
delete = navigateZS (Add (Const 1) (Neg (Var "a"))) $  
  do downZS  
    rightZS  
    r  $\leftarrow$  downZS  
    upZS  
    updateZS (const r)
```

*delete*  $\equiv$  *Just* (*Add* (*Const* 1) (*Var* "a"))

# 1. A zipper



The zipper we use is entirely standard:

**data** *Loc*  $\alpha$  **where**

*Loc*  $::$  (*Regular*  $\alpha$ , *Zipper* (*PF*  $\alpha$ ))  
 $\Rightarrow \alpha \rightarrow [\text{Ctx} (\text{PF } \alpha) \alpha] \rightarrow \text{Loc } \alpha$

**data family** *Ctx* ( $\phi :: \star \rightarrow \star$ )  $:: \star \rightarrow \star$

*enter*  $::$  (*Regular*  $\alpha$ , *Zipper* (*PF*  $\alpha$ ))  $\Rightarrow \alpha \rightarrow \text{Loc } \alpha$

*leave*  $::$  *Loc*  $\alpha \rightarrow \alpha$

*up*, *down*, *left*, *right*  $::$  *Loc*  $\alpha \rightarrow \text{Maybe} (\text{Loc } \alpha)$

*on*  $::$  *Loc*  $\alpha \rightarrow \alpha$

*updateM*  $::$  *Monad*  $\phi \Rightarrow (\alpha \rightarrow \phi \alpha) \rightarrow \text{Loc } \alpha \rightarrow \phi (\text{Loc } \alpha)$

# 1. Adding state to a zipper I



Now we embed this zipper in a state monad:

**type** *ZipperMonad*  $\alpha$   $\beta$  = *StateT* (*Loc*  $\alpha$ ) *Maybe*  $\beta$

Note that *ZipperMonad*  $\alpha$   $\beta \approx \text{Loc } \alpha \rightarrow \text{Maybe } (\beta, \text{Loc } \alpha)$ .

# 1. Adding state to a zipper I

Now we embed this zipper in a state monad:

**type** *ZipperMonad*  $\alpha$   $\beta$  = *StateT* (*Loc*  $\alpha$ ) *Maybe*  $\beta$

Note that *ZipperMonad*  $\alpha$   $\beta \approx \text{Loc } \alpha \rightarrow \text{Maybe } (\beta, \text{Loc } \alpha)$ .

Moving around:

*moveZS* :: (*Loc*  $\alpha \rightarrow \text{Maybe } (\text{Loc } \alpha)$ )  $\rightarrow$  *ZipperMonad*  $\alpha$   $\alpha$

*moveZS* *m* = *StateT* \$  $\lambda l \rightarrow m l \gg \lambda l' \rightarrow \text{return } (\text{on } l', l')$

*upZS*, *downZS*, *leftZS*, *rightZS* :: *ZipperMonad*  $\alpha$   $\alpha$

*upZS* = *moveZS* *up*

*downZS* = *moveZS* *down*

*leftZS* = *moveZS* *left*

*rightZS* = *moveZS* *right*

# 1. Adding state to a zipper II



Updating the value at the current location:

```
updateZS :: ( $\alpha \rightarrow \alpha$ )  $\rightarrow$  ZipperMonad  $\alpha$   $\alpha$ 
updateZS f = do l  $\leftarrow$  get
             let Just l' = updateM (Just  $\circ$  f) l
             put l'
             return (on l')
```

# 1. Another example



And that's all that's needed!






Another example, this time a swap:

```
swap :: Maybe Expr
swap = navigateZS (Add (Const 1) (Var "a")) $
  do l ← downZS
     r ← rightZS
     updateZS (const l)
     leftZS
     updateZS (const r)

swap ≡ Just (Add (Var "a") (Const 1))
```

# 1. Summary

A summary of the representation of transformations using a zipper with state:

-  Easy to use
-  Simple encoding
-  Suitable for structure editors
-  Transformations can be verbose
-  Transformations cannot be inspected

## 2. Rewriting



Our second approach uses a zipper to focus on a specific location, and rewrite rules to apply transformations:

```
insert :: Maybe Expr  
insert = apply [(down >>= >> right, addNeg)] ((Add (Const 1) (Var "a")))  
  where addNeg :: Rule Expr  
        addNeg = rule $  $\lambda x \rightarrow x \rightsquigarrow$  : Neg x  
  
insert  $\equiv$  Just (Add (Const 1) (Neg (Var "a")))
```

This is a combination of a zipper with generic rewrite rules (Van Noort et al., WGP 2008).



## 2. Meta-variable extension



Rewrite rules rely on meta-variable extension:

```
type Ext  $\phi$     = K Metavar :+:  $\phi$   
type Metavar = Int  
data  $\mu$   $\phi$       = In ( $\phi$  ( $\mu$   $\phi$ ))  
type Scheme  $\phi$  =  $\mu$  (Ext  $\phi$ )  
type SchemeOf  $\alpha$  = Scheme (PF  $\alpha$ )
```

## 2. More examples



Deleting a node:

*delete* :: *Maybe Expr*

*delete* = *apply* [(*down*  $\Rightarrow$  *right*, *removeNeg*)] *expr*<sub>2</sub> **where**

*removeNeg* :: *Rule Expr*

*removeNeg* = *rule* \$  $\lambda x \rightarrow \text{Neg } x \rightsquigarrow x$

## 2. More examples



Deleting a node:






```
delete :: Maybe Expr
delete = apply [(down >> right, removeNeg)] expr2 where
  removeNeg :: Rule Expr
  removeNeg = rule $ \x → Neg x :~::~ x
```

And here is the swap example:

```
swapExpr :: Rule Expr
swapExpr = rule $ \a b → Add a b :~::~ Add b a
```

## 2. Summary

A summary of the representation of transformations using rewrite rules:

-  Very easy to use
-  Very simple encoding
-  Suitable for exercise assistants
-  Not all sharing is captured
-  The representation of transformations is inconvenient:

*expandedSwap* :: *Rule Expr*

*expandedSwap* =

$$\begin{aligned} & \text{In} (R (R (R (R \\ & \quad (I (In (L (K 0))) :x: I (In (L (K 1))))))) \\ : \rightsquigarrow & \text{In} (R (R (R (R \\ & \quad (I (In (L (K 1))) :x: I (In (L (K 0))))))) \end{aligned}$$

### 3. Explicit encoding



Our third approach represents transformations using explicit paths to shared subterms:

*insert* :: *Maybe Expr*

*insert* = *apply addNeg (Add (Const 1) (Var "a"))* **where**

*addNeg* :: *Transformation Expr*

*addNeg* = *[[[1], Neg (Ref [1])]]*

### 3. Explicit encoding



Our third approach represents transformations using explicit paths to shared subterms:

*insert* :: *Maybe Expr*

*insert* = *apply addNeg (Add (Const 1) (Var "a"))* **where**

*addNeg* :: *Transformation Expr*

*addNeg* = *[([1], Neg (Ref [1]))]*

*delete* :: *Maybe Expr*

*delete* = *apply delNeg (Add (Const 1) (Neg (Var "a")))* **where**

*delNeg* :: *Transformation Expr*

*delNeg* = *[([1], Ref [1,0])]*

### 3. Explicit encoding



Our third approach represents transformations using explicit paths to shared subterms:

*insert* :: *Maybe Expr*

*insert* = *apply addNeg* (*Add* (*Const* 1) (*Var* "a")) **where**

*addNeg* :: *Transformation Expr*

*addNeg* =  $[[[1], \text{Neg} (\text{Ref } [1])]]$

*delete* :: *Maybe Expr*

*delete* = *apply delNeg* (*Add* (*Const* 1) (*Neg* (*Var* "a"))) **where**

*delNeg* :: *Transformation Expr*

*delNeg* =  $[[[1], \text{Ref } [1,0]]]$

*swap* :: *Maybe Expr*

*swap* = *apply swap'* (*Add* (*Const* 1) (*Var* "a")) **where**

*swap'* :: *Transformation Expr*

*swap'* =  $[[[0], \text{Ref } [1]], [1], \text{Ref } [0]]]$

### 3. Paths, references, and transformations



Paths are encoded as lists of integers:

```
type Path = [Int]
```

Yes, this is suboptimal.



### 3. Paths, references, and transformations



Paths are encoded as lists of integers:

```
type Path = [Int]
```

Yes, this is suboptimal.

References are encoded similarly to meta-variables for rewriting:

```
data WithRef  $\alpha$   $\beta$  = InR (PF  $\alpha$   $\beta$ )  
    | Ref Path
```

### 3. Paths, references, and transformations



Paths are encoded as lists of integers:

```
type Path = [Int]
```

Yes, this is suboptimal.

References are encoded similarly to meta-variables for rewriting:

```
data WithRef  $\alpha$   $\beta$  = InR (PF  $\alpha$   $\beta$ )  
    | Ref Path
```

Transformations are then lists of pairs containing paths and values extended with references:

```
type Transformation  $\alpha$  = [(Path,  $\mu$  (WithRef  $\alpha$ ))]
```

### 3. Ease of use



We get the same “ease of use” problem as with rewriting:

```
let  $expr_1 = \text{Add} (\text{Const } 1) (\text{Var } "a")$ 
```

```
 $insert :: \text{Maybe Expr}$ 
```

```
 $insert = \text{apply addNeg } expr_1 \text{ where}$ 
```

```
 $addNeg :: \text{Transformation Expr}$ 
```

```
 $addNeg = [([1], \text{In} \circ \text{InR} \circ \text{R} \circ \text{L} \circ \text{I} \circ \text{In} \$ \text{Ref } [1])]$ 
```

### 3. Ease of use



We get the same “ease of use” problem as with rewriting:

```
let  $expr_1 = \text{Add } (\text{Const } 1) (\text{Var } "a")$ 
```

```
 $insert :: \text{Maybe Expr}$ 
```

```
 $insert = \text{apply } addNeg \ expr_1 \ \mathbf{where}$ 
```

```
 $addNeg :: \text{Transformation Expr}$ 
```

```
 $addNeg = [([1], In \circ InR \circ R \circ L \circ I \circ In \$ Ref [1])]$ 
```

We solve it by adding a layer of Template Haskell for convenience:

```
 $insert :: \text{Maybe Expr}$ 
```

```
 $insert = \text{apply } (\text{fromTransformation}_{EH} \ addNeg) \ expr_1 \ \mathbf{where}$ 
```

```
 $addNeg :: \text{Transformation}_{EH} Expr$ 
```

```
 $addNeg = [([1], Neg_{EH} (Ref_{EH} [1]))]$ 
```

### 3. Generic diff



Unlike the previous approaches, this time we have a  $\text{diff} :: (\text{Regular } \alpha, \dots) \Rightarrow \alpha \rightarrow \alpha \rightarrow \text{Transformation } \alpha$  such that:





$$\forall a, b. \text{apply } (\text{diff } a \ b) \ a \equiv \text{Just } b$$

Our  $\text{diff}$  returns *Transformations* with maximal use of sharing (but other algorithms are possible). More details in the paper.

### 3. Summary



A summary of the explicit representation of transformations:

-  Sharing is explicit
-  We can provide *diff*
-  Transformations are easy to inspect
-  Interface is only convenient through Template Haskell

## Summarising:

- ▶ Representing transformations effectively is important
- ▶ We show three different ways of doing it
- ▶ All generic, working on families of datatypes (using `multirec`)
- ▶ A nice combination of multiple generic programming techniques

Summarising:

- ▶ Representing transformations effectively is important
- ▶ We show three different ways of doing it
- ▶ All generic, working on families of datatypes (using `multirec`)
- ▶ A nice combination of multiple generic programming techniques

All the code is available at

<http://hackage.haskell.org/package/transformations>.



Summarising:

- ▶ Representing transformations effectively is important
- ▶ We show three different ways of doing it
- ▶ All generic, working on families of datatypes (using `multirec`)
- ▶ A nice combination of multiple generic programming techniques

All the code is available at  
<http://hackage.haskell.org/package/transformations>.

Questions?